Streaming potential effects on solute dispersion in nanochannels

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This Supporting Information includes the derivation of the pressure-driven flow induced streaming potential field, eq 12, and the derivation of the electrokinetic "figure of merit", eq 15.

Combining equations 2 and 11 yields the electric current density j in electrokinetic flow along a slit channel,

$$j = -\frac{\varepsilon RT}{2\mu F} \frac{d^2 \Psi}{dy^2} \left(1 - y^2\right) P + \frac{\varepsilon^2 \zeta RT}{a^2 \mu F} \frac{d^2 \Psi}{dy^2} \left(1 - \frac{\Psi}{\zeta^*}\right) E + c_b \lambda_b \cosh(\Psi) E \,. \tag{S1}$$

The zero electric current in a steady-state pressure-driven flow forces

$$0 = -\frac{\varepsilon RT}{2\mu F} \int_0^1 \frac{d^2 \Psi}{dy^2} (1 - y^2) dy P + \frac{\varepsilon^2 \zeta RT}{a^2 \mu F} \int_0^1 \frac{d^2 \Psi}{dy^2} (1 - \frac{\Psi}{\zeta^*}) dy E_{st} + c_b \lambda_b \int_0^1 \cosh(\Psi) dy E_{st}.$$
 (S2)

Therefore, the induced streaming potential field is determined as

$$E_{st} = \frac{\frac{\varepsilon RT}{2\mu F} \int_0^1 \frac{d^2 \Psi}{dy^2} (1 - y^2) dy}{\frac{\varepsilon^2 \zeta RT}{a^2 \mu F} \int_0^1 \frac{d^2 \Psi}{dy^2} \left(1 - \frac{\Psi}{\zeta^*}\right) dy + c_b \lambda_b \int_0^1 \cosh(\Psi) dy} P.$$
(S3)

Employing the integration by parts rewrites eq S3 as

$$E_{st} = \frac{\frac{\varepsilon RT\zeta^*}{\mu F}g_1}{\frac{\varepsilon^2 R^2 T^2}{a^2 \mu F^2} \int_0^1 \left(\frac{d\Psi}{dy}\right)^2 dy + c_b \lambda_b g_3} P$$
(S4)

where the functions g_1 and g_3 are as defined in the main text. Multiplying the two sides of the Poisson-Boltzmann equation by $2d\Psi/dy$ and then integrating the new eq 3 leads to

$$\left(\frac{d\Psi}{dy}\right)^2 = 2\kappa^{*2} \left[g_3 - \cosh\left(\Psi_0\right)\right].$$
(S5)

Thus, eq S4 can be simplified to eq 12 after mathematical manipulations, wherein the expression $\kappa^{*2} = 2a^2 F^2 c_b / \varepsilon RT$ has been invoked.

As for the definition of electrokinetic "figure of merit", we first get the following relationship by area-averaging the two fluid velocity components in eq 2

$$\frac{\langle u_e \rangle}{\langle u_p \rangle} = -\frac{3\varepsilon\zeta}{a^2} g_1 \frac{E_{st}}{P}.$$
(S6)

Then, replacing E_{st} with eq 12 and invoking $\kappa^{*2} = 2a^2 F^2 c_b / \varepsilon RT$ again receive

$$\frac{\langle u_e \rangle}{\langle u_p \rangle} = -\frac{3\zeta^{*2}g_1^2}{2\phi^2 [(1+\beta/4)g_3 - \cosh(\Psi_0)]} = -Z .$$
(S7)